REVIEW
of the official reviewer for the dissertation work of
Nauryz Targyn Atanbekovich on the topic "The method of heat polynomials and special functions for the problem of heat equation in regions with free boundaries and their application", submitted for the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 - Mathematical and computer modeling".

№ p/p	Criteria	Compliance with the criteria (you must mark one of the answer options)	Justification of the official reviewer's position
1.	The topic of the dissertation (as of the date of its approval) corresponds to the directions of development of science and / or state programs	1.1 Compliance with priority areas for the development of science or government programs:	Fundamental and applied research in mathematics and physics
		1) The dissertation was completed within the framework of a project or target program financed from the state budget (indicate the name and number of the project or program) 2) The dissertation was completed within the framework of another state program (indicate the name of the program) **3) The dissertation corresponds to the priority direction of the development of science, approved by the Higher Scientific and Technical Commission under the Government of the Republic of Kazakhstan (indicate the direction)**	Project AP09258948 "Problems with a free boundary in mathematical models of electrical contact phenomena"
2.	Importance for science	The work **makes** / does not make a significant contribution to science, and its importance is well disclosed / not disclosed	The results and conclusions represent a significant contribution to the mathematical modeling of thermophysical processes in electrical contact phenomena.
3.	The principle of independence	Self-reliance level: 1) **High;** 2) Medium; 3) Low;	Most of the results of the dissertation, in particular, the analytical solution of the problem, finding unknown coefficients in a linear combination of special

		4) There is no independence	functions, proving their convergence, numerical results, proving the existence and uniqueness of the solution, have been obtained and show a high level of independence in the performance of work
4.	The principle of internal unity	4.1 Rationale for the relevance of the dissertation: 1) **Justified;** 2) Partially justified; 3) Not substantiated.	The work is aimed at studying thermal processes in electrical contacts, which are described by the heat equation with free boundaries. In the first part of the dissertation, thermal polynomials and special functions are introduced. The second part studies direct and inverse Stefan problems in spherical domains and regions with a variable cross section. In the third part, the spherical Stefan problem with non-linear thermal coefficients is studied where a similar solution is found and their existence and uniqueness are proved.
		4.2 The content of the dissertation reflects the topic of the dissertation: 1) **reflects;** 2) partially reflects; 3) does not reflect	The topic of the dissertation is reflected in the content and gives a complete understanding of the meaning of the dissertation
		4.3. The purpose and objectives correspond to the topic of the dissertation: 1) **correspond;** 2) partially correspond; 3) do not correspond	The main goal of the dissertation is to develop new exact and approximate analytical methods for solving heat and mass transfer problems. The main objectives of the work: - Studying spherical Stefan problem with heat polynomials; - Studying Stefan problem in regions with variable cross-section and cylindrical domain with special functions; - Nonlinear Stefan problem in

			spherical region and similarity type solution
		4.4 All sections and provisions of the dissertation are logically interconnected: 1) **are fully interconnected;** 2) the relationship is partial; 3) there is no relationship	The main sections of the dissertation are closely related, in particular, the first and second sections are fully interconnected, they characterized completely the method of heat polynomials and special functions, their effectiveness in solution of the problems related to electrical contact phenomena. The third section is interconnected to spherical Stefan problem considered in the second section and difference is that thermal coefficients are depended on temperature.
		4.5 The new solutions proposed by the author (principles, methods) are argued and evaluated in comparison with the known solutions: 1) **there is a critical analysis;** 2) partial analysis; 3) the analysis is not one's own opinions, but quotes from other authors	The author presented accurate, logically verified substantiations of the main provisions of the dissertation submitted for defense
5.	The principle of scientific novelty	5.1 Are scientific results and provisions new? 1) **completely new;** 2) partially new (25-75% are new); 3) not new (less than 25% are new)	The main results of the dissertation are obtaining an analytical solution of the problem of heat conduction using the method of thermal polynomials, special functions, proof of their convergence. Another important result obtained in this paper is the existence and uniqueness of such a solution to the non-linear Stefan problem, which has attracted increased attention in recent years
		5.2 Are the conclusions of the dissertation new? 1) completely new; 2) **partially new (25-75% are new);** 3) not new (less than 25% are new)	The conclusions made by the author are new: - Obtaining a numerical solution of the inverse spherical Stefan problem based on the method of thermal polynomials and the variational method,

			- Analytical solution of the direct Stefan problem in bodies with variable cross section based on the method of special functions - Finding the heat flux entering the electrodes using a linear combination of thermal polynomials and special functions -Existence and uniqueness of a similar type of solution to the Stefan problem with enhanced nonlinearity
		5.3 Technical, technological, economic or managerial decisions are new and justified: 1) completely new; 2) **partially new (25-75% are new);** 3) not new (less than 25% are new)	Technical, technological, economic or managerial decisions are partially new. Obtained results can be used in electrical contact systems in low-voltage devices
6.	Validity of the main conclusions	All key findings are **based**/not based on scientifically sound evidence, or reasonably well-founded (for qualitative research and arts and humanities courses)	The main conclusions of the thesis are fully justified
7.	Basic provisions for defense	The following questions need to be answered for each position separately: 7.1 Is the position proven? 1) **proven;** 2) rather proven; 3) rather unproven; 4) not proven 7.2 Is it trivial? 1) **yes;** 2) no 7.3 Is it new? 1) **yes;** 2) no 7.4 Level to apply: 1) narrow; 2) **medium;** 3) wide 7.5 Is it proven in the article?	The following results are presented for defense: - Numerical solution of the inverse spherical Stefan problem by the method of thermal polynomials and variational method; - Analytical exact solution of the Stefan problem in bodies with variable cross sections based on the method of special functions and their convergence - Proof of the existence and uniqueness of a similar solution to the spherical Stefan problem with thermal coefficients depending on temperature; -Using the result for mathematical modeling of thermal processes in a short arc with spontaneous rejection of

		1) **yes;** 2) no	electrical contacts, during the transition of the metalic phase of the arc to the gaseous phase
8.	The principle of certainty Reliability of sources and information provided	8.1 The choice of methodology is justified or the methodology is described in sufficient detail 1) **yes;** 2) no	The author makes good use of the proven methods of the theory of partial differential equations
		8.2 The results of the dissertation work were obtained using modern methods of scientific research and methods of processing and interpreting data using computer technologies: 1) **yes;** 2) no	Numerical and graphical results of the analytical solutions of the Stefan problems considered in dissertation obtained by computer program Mathcad
		8.3 Theoretical conclusions, models, identified relationships and patterns are proven and confirmed by experimental research (for areas of training in pedagogical sciences, the results are proven on the basis of a pedagogical experiment): 1) **yes;** 2) no	Theoretical conclusions are proven and successfully discussed
		8.4 Important statements are **supported** / partially confirmed / not supported by references to relevant and reliable scientific literature	The results of the work are consistent with previously published works. Exact references to related sources are provided
		8.5 Used literature sources are **sufficient** / not sufficient for a literature review	Quantity of literature satisfied, but references to sources could be expanded
9	Principle of practical value	9.1 The dissertation has a theoretical value: 1) **yes;** 2) no	The dissertation has theoretical value
		9.2 The dissertation is of practical importance and there is a high probability of applying the results obtained in practice: 1) **yes;** 2) no	The results of dissertation can find application in electrical contact devices with low voltage
		9.3 Are the practice suggestions new? 1) completely new; 2) **partially new (25-75% are new);** 3) not new (less than 25% are new)	There are partially new recommendations that may be useful in practice

10.	Quality of writing and design	Quality of academic writing: 1) high; 2) **medium** 3) below medium; 4) low.	The style of the dissertation is medium

Conclusion:

The dissertation work of Nauryz Targyn Atanbekovich on the topic "The method of heat polynomials and special functions for the problem of heat equation in regions with free boundaries and their application" submitted for the degree of Doctor of Philosophy (PhD) meets all the qualification requirements of the Rules for awarding degrees, presented for doctoral (PhD) dissertations. I consider Nauryz Targyn Atanbekovich to be awarded the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 – Mathematical and computer modeling".

Official reviewer:

Doctor of physical and mathematical sciences,
professor, distinguished professor
of the Karaganda University
named after academician Buketov

(signature)

Ramazanov M.I.